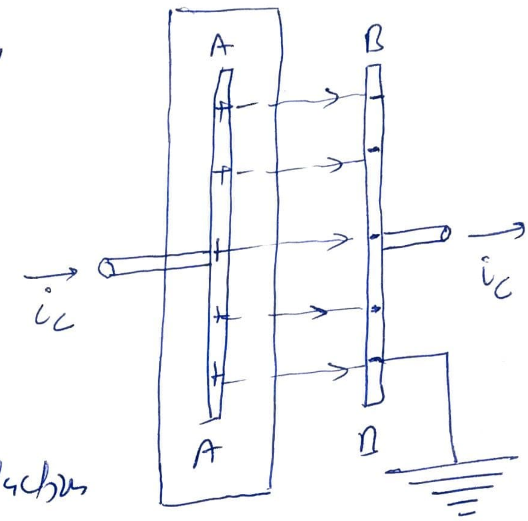
Displacement Current

Kirchoff's Law → The sum of incoming currents on any point in a steady state close circuit is equal to the sum of outgoing currents from that point.

This law does not apply to a Capacitor being charged.



Current i_c enters in plate AA, but no current coming out. Similarly conduction current i_c coming out from plate BB, but no current is entering.

According to Maxwell → When a capacitor is charged the charge on its plate increases slowly due to conduction current. The intensity of electric field between the plates of capacitor increases with increase in charge. The rate of increase of intensity of electric field was ~~defined~~ related to equivalent current density.

If σ → uniform charge density on the plates of capacitor

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} \quad \text{——— (1)}$$

If dQ → increase in charge Q in time 'dt'

$$dE = \frac{dQ}{\varepsilon_0 A} = \frac{I\,dt}{\varepsilon_0 A} \quad \text{——— (2)}$$

I → Conduction current

$$\frac{dE}{dt} = \frac{I}{\varepsilon_0 A} \quad \text{---(3)} \qquad \frac{I}{A} = \frac{\partial D}{\partial t}$$

If we define the equivalent current density J_D between the plates;

$$J_D = \varepsilon_0 \frac{dE}{dt} \quad \text{---(4)}$$

The total equivalent current between the plates I_D, $I_D = J_D A$

$$I_D = J_D A = \varepsilon_0 \frac{dE}{dt} A \quad \text{---(5)}$$

From eqn (3) & (5) $\underline{I_D = I}$

This equivalent current $\rightarrow$ Displacement Current.

The magnetic field is produced by this current is similar way as conduction current $\cancel{I}_C$.

$\frac{I}{A} = \frac{\partial D}{\partial t} \rightarrow$ density of some current which correspond, to the "current" which flow in the space, even in vacuum, between the pair of plates of a capacitor when the charged plates are connected by a wire, thus completing the conduction current

Idea of the relative size of the two types of current in conductors

Consider a copper wire in which electric field

$$E = E_0 e^{-i\omega t} \qquad j = \sigma E = \sigma E_0 e^{-i\omega t}$$

$$\Rightarrow |j|^2 = \sigma^2 |E_0|^2$$

and $D = \varepsilon_0 E_0 e^{-i\omega t}$

$$\frac{\partial D}{\partial t} = -i\omega \varepsilon_0 E_0 e^{-i\omega t}$$

$$\left|\frac{\partial D}{\partial t}\right|^2 = \omega^2 \varepsilon_0^2 |E_0|^2 \quad \Rightarrow \quad \left|\frac{j}{\frac{\partial D}{\partial t}}\right| = \frac{\sigma}{\omega \varepsilon_0} \text{ for copper}$$

$\sigma = 5.9 \times 10^7$. $\Rightarrow \frac{\sigma}{\omega \varepsilon_0} \sim \frac{10^{19}}{\omega} \rightarrow$ ratio is very large for all frequencies.

Ampere's Law and Maxwell's Modification

Ampere's Law: The line integral of magnetic field is equal to μ_0 times the I_{enc} current in the closed surface

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} \quad \text{---(1)}$$

The magnetic field can be established by

(I) Conduction current

(II) Changing electric field or displacement current

from Faraday's Law

$$e_{ind} = \oint \vec{E}\cdot d\vec{l} = (-)\frac{d\phi_B}{dt}$$

Similarly, relation of changing electric flux ψ and magnetic field (B)

$$\oint B\cdot dl = \mu_0 \varepsilon_0 \frac{d\psi}{dt}$$

So Ampere's Law

$$\oint B\cdot dl = \mu_0 \varepsilon_0 \frac{d\psi}{dt} + \mu_0 I_{enc} \quad \text{---(2)}$$

Electric flux $\psi = \int \vec{E}\cdot\vec{ds}$

$$\frac{d\psi}{dt} = \frac{d}{dt}\left[\int \vec{E}\cdot\vec{ds}\right]$$

$$= \int \left[\frac{d\vec{E}}{dt}\cdot\vec{ds}\right]$$

From eq^n (2)

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \varepsilon_0 \int \left(\frac{d\vec{E}}{dt}\right)\cdot ds + \mu_0 I_{enc}$$

Modified Ampere's Law

Maxwell eqn for time dependent ~~El~~ electromagnetic field in Vacuum

Gauss Law $\oint E.ds = \frac{1}{\varepsilon_0} \Sigma q_i \Rightarrow \oint \vec{E}.\vec{ds} = \frac{1}{\varepsilon_0} Q \ ; \ Q = \Sigma q_i$

From divergence theorem

$$\oint \vec{E}.\vec{ds} = \int div\, \vec{E}\, dv$$

$$= \int \nabla.E\, dV \quad \text{---(1)}$$

$Q = \int_V \rho\, dV$

$$\Rightarrow \int \nabla.\vec{E} dV = \frac{1}{\varepsilon_0} \int \rho\, dV$$

$$\therefore \boxed{\vec{\nabla}.\vec{E} = \frac{1}{\varepsilon_0} \rho} \quad \text{---(2)}$$

In vacuum $\boxed{\vec{\nabla}.\vec{B} = 0}$ --- (3)

Integral form of Faraday's Law

$$\int \vec{E}.\vec{dl} = (-) \frac{d\phi_B}{dt} = (-) \int \frac{d\vec{B}}{dt}.\vec{ds}$$

From Stoke's theorem

$$\oint \vec{E}.\vec{dl} = \oint \vec{\nabla}\times\vec{E}.ds$$

$$\Rightarrow \oint \vec{\nabla}\times\vec{E}.ds = (-) \int \frac{d\vec{B}}{\partial t}.\vec{ds}$$

$$\Rightarrow \boxed{\nabla\times\vec{E} = -\frac{dB}{\partial t}} \quad \text{---(4)}$$

The Modified Ampere's Law

$$\int B.dl = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \int \frac{\partial \vec{E}}{\partial t}.\vec{ds}$$

from Stoke's theorem $\int \vec{B}.\vec{dl} = \int \vec{\nabla}\times\vec{B}.ds$

$I_{enc} = J.ds$

$$\Rightarrow \boxed{\vec{\nabla}\times\vec{B} = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}} \quad \text{---(5)}$$

Maxwell's eqⁿ in vacuum (Differential form)

(i) $\vec{\nabla}\cdot\vec{E} = \frac{1}{\varepsilon_0}\rho$ or $\vec{\nabla}\cdot\vec{D} = \rho$

(ii) $\vec{\nabla}\cdot\vec{B} = 0$ or $\vec{\nabla}\cdot\vec{H} = 0$

(iii) $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$ or $\vec{\nabla}\times\vec{E} + \mu_0\frac{\partial H}{\partial t} = 0$

(iv) $\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$ or $\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$

Integral form

$$\oint \vec{D}\cdot d\vec{s} = \int_V \rho\, dv$$

$$\oint \vec{B}\cdot d\vec{s} = 0$$

$$\oint \vec{E}\cdot d\vec{l} = (-)\int \frac{\partial\vec{B}}{\partial t}\cdot d\vec{s}$$

$$\oint \vec{H}\cdot d\vec{l} = I + \int \frac{\partial\vec{D}}{\partial t}\cdot d\vec{s}$$

Maxwell eqⁿ in Matter (Dielectric)

if $\rho = 0$ in medium (dielectric)

(i) $\vec{\nabla}\cdot\vec{E} = 0$ (electrostatics)

(ii) $\vec{\nabla}\cdot\vec{B} = 0$

(iii) $\vec{\nabla}\times\vec{E} = (-)\frac{\partial\vec{B}}{\partial t}$

(iv) $\vec{\nabla}\times\vec{B} = \mu\varepsilon\left(\frac{\partial\vec{E}}{\partial t}\right)$

Boundary Conditions : We can obtain boundary conditions from Maxwell's eq^n (Integral form)

The tangential component of Electric field across interface between two medium

$$E_{t_1} = E_{t_2}$$ and for magnetic field $H_{t_1} = H_{t_2}$. From surface Integral of the eq^n, the normal component boundary conditions are

$$D_{n_1} - D_{n_2} = \rho_s \quad \text{and} \quad B_{n_1} = B_{n_2}$$

In Ideal conditions, (i) Ideal conductor $\sigma = \infty$, current density $J \to$ limited, $\vec{E} = 0$, from Faraday's Law for time varying fields $H = 0$, from Ampere's Circuital law, the limited value of current density $J = 0$, the current in conductor will be surface current, k.

If one of the medium is Ideal conductor, the boundary conditions will be,

$$E_{t_1} = 0, \quad H_{t_1} = k \quad \text{where} \quad \vec{H}_{t_1} = k \times \hat{a}_n$$

$$D_{n_1} = \rho_s, \quad B_{n_1} = 0$$

Prob. A car ignition coil consists of two insulated coils, one of 160000 turns and the other of 4000 turns, wound over each other. The length of each coil is 10 cm and the turns have the radius of 3 cm. A current of 3 A is passed through the primary coil and broken in about 10^{-4} sec. Calculate the voltage induced in the secondary circuit.

Sol^n The magnetic flux density in the solenoid

$$B = \mu_0 N_1 I_1 \qquad N_1 \to \text{No. of turns per unit length}$$
$$I_1 \to \text{Current.}$$

The magnetic flux through each turn of the top coil is $\mu_0 N_1 I_1 \pi r^2$ and total flux is

$$\phi_2 = \mu_0 N_1 N_2 l \pi r^2 I_1$$

Induced emf $\quad \mathcal{E} = -\frac{d\phi_2}{dt} = -\mu_0 N_1 N_2 l \pi r^2 \frac{dI_1}{dt}$

$$\frac{dI_1}{dt} = 3 \times 10^{+4} \, As^{-1} \qquad = -M \frac{dI_1}{dt}$$

$$\mathcal{E} = 4\pi \times 10^{-7} \times 16 \times 10^4 \times 4 \times 10^3 \times 10^{-1} \pi \times (0.03)^2 \times 3 \times 10^4$$
$$= 6314 \text{ Volts}$$